

# Experimental dynamic validation of a numerical model of a CRP cylindrical shell

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**ABSTRACT:** This paper presents the results of some experimental dynamic tests carried out on a CRP multi-layer cylindrical shell with the main purpose of validating a previously derived finite element model. This model has been formulated assuming, for each layer, homogeneous and orthotropic behavior in elastic regime, while the material properties have been set up according to the results of previous static laboratory tests. Experimental and numerical modal properties (natural frequencies, modal damping ratios, modal shapes) have been correlated and compared, observing a reasonable agreement between the first natural frequencies and the first three couples of flexural modal shapes. However the second and the third numerical natural frequencies have been found considerably greater than the experimental ones, suggesting a model updating which will be dealt with in a subsequent paper.

## 1 INTRODUCTION

Composite materials, like CRP (Carbon Reinforced Polymer) materials, are nowadays largely used for a very wide range of mechanical and civil engineering products. This is due to their lightweight and high-strength characteristics, together with the capabilities of new technologies to provide a wide range of different profiles. However these products are still more expensive than those obtained through more traditional materials, i.e. metals, indeed because their design is mainly based on tests carried out on real prototypes. In this context the availability of accurate numerical models to be used as design tools becomes of utmost importance. On the other hand, the formulation of such models requires the knowledge of the material mechanical properties, which can be effectively derived only by means of appropriate laboratory static and/or dynamic tests.

However, not all the required experimental investigations can be easily performed, especially in the static field. As a matter of fact an accurate estimation of shear moduli by means of static tests is very difficult to achieve. For these reasons a correct strategy to predict the mechanical behavior of a structural element should be based on the employment of both static and dynamic tests as well as on the use of an accurate numerical model.

Generally, the aim of dynamic tests is twofold: (i) to determine the nature and the extent of vibration response levels under operational conditions and (ii) to verify and update, if necessary, theoretical mod-

els. Excessive vibration levels can indeed determine temporary malfunction or can create disturbance or discomfort, so they have to be minimized. Therefore, the vibration levels in service or in operational conditions should be known in advance with a sufficient degree of confidence and brought under satisfactory control.

Furthermore, some essential material properties can also be measured under dynamic loads, such as both elastic and viscoelastic properties, damping capacity and fatigue endurance.

Recent years have seen a growing interest in using dynamic experimental techniques for the characterization of composite materials, especially in the field of automotive, aerospace, high performance structures and machines (Gibson 2000). In fact, the importance of the accurate knowledge of the damping properties has been recognized in the design process, as well as the control of noise and vibration in high precision devices. Furthermore, improvements in quality control can be achieved by measuring material dynamic properties during manufacturing.

This paper presents some preliminary results of an ongoing research, dealing with the structural identification and the mechanical characterization of CRP structural elements. Some dynamic tests on a multilayer cylindrical shell have been carried out under impulsive excitations in “free-free” boundary conditions. The dynamic response has been measured by a set of accelerometers and the experimental modal analysis has been performed in the time

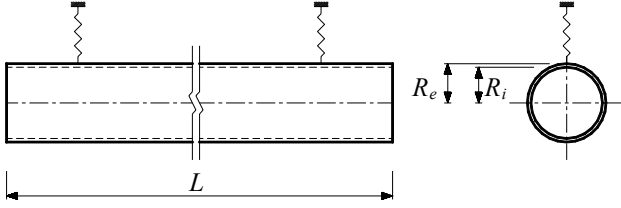


Figure 1. Lateral view and cross section of the cylindrical shell.

domain by means of the Ibrahim identification technique (Ibrahim & Mikulcik 1976, 1977, Pappa & Ibrahim 1981). The experimental modal properties (natural frequencies, modal damping factors and modal shapes), have been correlated and compared with those obtained by means of a FE model, in which the material properties have been set up according to a series of static experimental tests (Dispenza et al. 2002a, b). A good match has been found between the first experimental and numerical natural frequencies, as well as among the first three couples of flexural modal shapes. Conversely, the second and the third natural frequencies evaluated through the FE model are considerably higher than the corresponding experimental estimates. This suggests that the model should be updated, and this subject will be dealt with in an incoming paper.

## 2 THE STRUCTURAL ELEMENT

The structural element under investigation is the CRP multi-layer cylindrical shell shown in Figure 1, having length  $L = 1500$  mm, external radius  $R_e = 40$  mm, thickness  $d = 2$  mm, and mass density  $\rho = 1.45 \times 10^{-9}$  Nsec<sup>2</sup>/mm<sup>4</sup>. The tube is made of carbon-fiber, epoxy-resin composite material and manufactured by means of a pull-winding technology. Five layers of fibers have been employed, each characterized by a different winding angle with respect to the longitudinal axis of the tube. In the following, these layers are progressively numbered from the outer to the inner diameter.

The mechanical characteristics of the tube are affected by the location of the fibers along the wall thickness of the pole, by the value of the winding angle and by the amount of fibers in each layer. After some preliminary studies among different laminate lay-ups (Dispenza et al. 2002b) the one reported in Table 1 has been chosen. The last row of Table 1 reports the mean value of the fiber volume fraction for each layer, as measured by Dispenza et al. (2002a), which are essential to analytically deter-

Table 1. Geometric characteristics of the tube for each layer.

Layer number		1	2	3	4	5
Winding angle	(°)	0	75	0	-75	0
Thickness	(mm)	0.39	0.28	0.77	0.28	0.28
Fibre volume	(%)	52	42	59	42	59

Table 2. Longitudinal apparent moduli for each layer.

Layer	$E_a$ (GPa)	$E_b$ (GPa)	$E_c$ (GPa)
1	6.16	124	6.16
2-4	5.12	100	5.12
3-5	7.18	140	7.18

mine the Young's moduli of the laminate.

Each layer of the composite material was modeled like a homogeneous and orthotropic material, so that its mechanical properties in elastic regime are related to the knowledge, along the orthotropic directions, of the longitudinal and transverse apparent moduli  $E_a, E_b, E_c, G_{ab}, G_{ac}, G_{bc}$  ( $a, b, c$  being the "material axes", with  $b$  tangent to the fibers and  $a$  and  $c$  orthogonal to the fibers) as well as of the Poisson's ratios,  $\nu_{ab}, \nu_{ac}, \nu_{bc}$  along the same directions. Moreover, considering the variation of fiber volume fractions in each layer, the above mentioned quantities change from layer to layer.

The longitudinal apparent moduli, reported in Table 2, have been obtained following the procedure described in Dispenza et al. (2002a). The transverse apparent moduli and Poisson's ratios have been considered equal for all the layers and, according to Fuschi et al. (2003), have been set equal to:  $G_{ab} = 1.35$  GPa,  $G_{ac} = G_{bc} = 1.00$  GPa,  $\nu_{ab} = 0.30$ ,  $\nu_{ac} = \nu_{bc} = 0.015$ .

## 3 FREE VIBRATION TESTS

### 3.1 Experimental set up

The specimen has been tested in "free-free" boundary conditions, which have been simulated hanging the test object by means of two soft springs. In order to minimize the influence of the suspension on the lowest bending mode of the specimen, the springs have been attached to the nodal points of the fundamental vibration mode of a prismatic, homogeneous, unconstrained beam. Experimental results confirmed that these points are really very close to

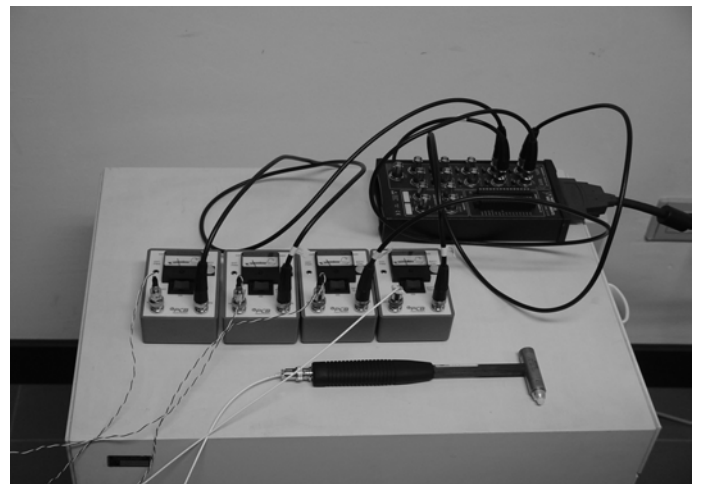


Figure 2. 086C03 PCB impact hammer and 480C02/480C PCB battery signal conditioning units.



Figure 3. Test object, grid of measurement points, suspension system and M353B17 PCB piezoelectric accelerometers located on a straight line.

the ones of the analyzed structural element. A grid of measurement points has been set along the specimen, at intervals of 10.0 cm.

The dynamic tests have been performed under impulsive excitations. The impulsive forces have been provided by a 086C03 PCB impact hammer, with sensitivity equal to 3500 mV/N, Figure 2. The free vibration response has been recorded by means of three M353B17 PCB piezoelectric accelerometers with frequency range from 10 up to 10000 Hz, measurement range  $\pm 500$  g pk and average sensitivity 10.42 mV/g.

The accelerometers have been attached to the specimen by a thin layer of wax according to two different patterns. Dynamic tests have been carried out with the accelerometers located either radially, at the mid-span cross section, or on a straight line along the test object, Figure 3. In both cases, the impact points have been moved over the specimen, in order to obtain three rows of the frequency response function matrix.

The analogue signals have been converted in digital data and recorded by means of a 16 bit 6052E National Instruments A/D converter. The resolution of the acquisition system is equal to  $2.92 \times 10^{-3}$  g.

In all the tests, data have been recorded for a time interval of 4 sec. with a sampling rate of 5000 Hz.

### 3.2 Data processing

Aiming at minimizing errors due to leakage, the recorded signals have been multiplied by an “exponential window” in order to have input signals which are nearly zero at the end of the recording time. The rigid body motion frequencies are lower than the investigated frequency range. However, each signal has been filtered by a high-pass filter with a cut-off frequency of 200 Hz. Furthermore, since the impulsive forces are of short duration, the noise during the rest of the signals has been removed by means

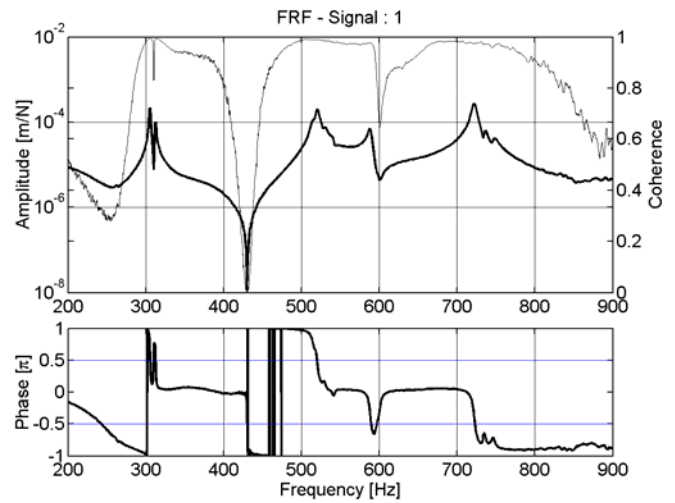


Figure 4. Typical average frequency response function in terms of displacement recorded at a quarter of the span.

of a suitable “force window”.

The input and output signals have been transformed in the frequency domain by using the fast Fourier transform (FFT) algorithm. Considering the good degree of consistency of the measurements, the frequency response functions at each measurement station have been evaluated by making the average of five impulsive tests. Figure 4 shows a typical average frequency response function together with the corresponding coherence function. It can be noted that this latter exhibits values close to one near the peaks, which assures a good correlation of the recorded signals around the resonances.

### 3.3 Modal parameter extraction

The dynamic characteristics of the test object have been extracted from a set of average impulse response functions by means of the Ibrahim Time Domain (ITD) identification technique (Ibrahim & Mikulcik 1976, 1977, Pappa & Ibrahim 1981). This method leads to a set of modal parameters (i.e. natural frequencies, modal damping ratios and mode shapes) in a single analysis even in the case of strong modal coupling and/or vibration modes with a relatively small contribution in the response.

The average impulse response functions have been obtained applying the inverse FFT (IFFT) algorithm to the average frequency response functions.

The physical modes have been distinguished from the computational ones by means of the overall modal confidence factor (OAMCF) (Ibrahim 1978) and the mode shape coherence and confidence factor (MSCCF) (Gao & Randall 2000).

Figure 5 shows the stabilization diagram in which the curve on the background represents the sum of the average frequency response functions.

In this diagram the best estimates of the natural frequencies are reported as functions of the assumed number of degrees of freedom, i.e. the number of pseudo-measurements taken into account during the

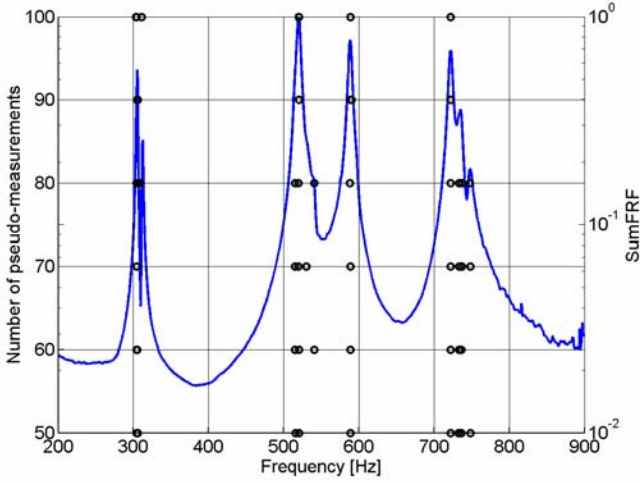


Figure 5. ITD stabilization diagram.

identification procedure.

Five natural frequencies have been identified in the range from 200 to 900 Hz. The average values of these frequencies are reported in Table 3 together with the corresponding modal damping ratios. It can be noted that all modal damping ratios are less than 1%.

#### 4 COMPARISON WITH THE RESULTS OF A FINITE ELEMENT MODEL

Experimental results have been compared with the numerical ones produced by the previously recalled model.

In particular a modal analysis has been executed by means of the FE code ADINA 8.0 (2003) using the Lanczos iteration method. This kind of analysis allows one to obtain a numerical evaluation of the undamped frequencies and modal shapes of the structural element under investigation.

The multi-layer composite tube has been modeled making use of 9-nodes isoparametric multi-layered shell elements. The utilized mesh presents 3200 five-layered shell elements and has been obtained by 40 subdivisions along the hoop direction and by 80 subdivisions along the longitudinal direction.

As far as the material is concerned, each layer has been characterized by means of the geometrical and material parameters reported in Section 2. Moreover the constraint conditions have been reproduced considering a series of springs located as specified in Section 3.1.

The comparison between experimental and numerical results has been carried out considering that the available data acquisition system allows to esti-

Table 3. Identified average natural frequencies and modal damping ratios.

	1	2	3	4	5
$f$ (Hz)	304.65	311.79	519.90	588.25	723.03
$\xi$ (%)	0.20	0.18	0.42	0.40	0.37

Table 4. First four flexural numerical undamped frequencies.

	1	2	3	4
$f$ (Hz)	319.00	698.15	1092.37	1470.78

mate only flexural modal shapes. Bearing this in mind, only the first four flexural undamped natural frequencies obtained from the finite element analysis have been taken into consideration. These frequencies are reported in Table 4.

For the sake of completeness, it should be noted that the FE model provides other seven frequencies in the range from 200 to 900 Hz, equal to 486.8, 531.4, 532.7, 537.9, 588.2, 710.4 and 884.5 Hz respectively. However these frequencies have not been compared with experimental ones because their corresponding modal shapes are essentially associated to transverse vibrations of cross sections, where the longitudinal axis remains straight.

To evaluate the correlation among experimental and numerical modal shapes the modal assurance criterion (MAC) (Ewins 2000, Haylen et al. 1999) has been employed, which is defined as

$$MAC(\boldsymbol{\varphi}_{exp}^{(i)}, \boldsymbol{\varphi}_{num}^{(j)}) = \frac{|\boldsymbol{\varphi}_{exp}^{(i)T} \boldsymbol{\varphi}_{num}^{(j)}|^2}{|\boldsymbol{\varphi}_{exp}^{(i)T} \boldsymbol{\varphi}_{exp}^{(i)}| |\boldsymbol{\varphi}_{num}^{(j)T} \boldsymbol{\varphi}_{num}^{(j)}|} \quad (1)$$

where  $\boldsymbol{\varphi}_{exp}$  and  $\boldsymbol{\varphi}_{num}$  refer to experimental and numerical modal shapes respectively,  $T$  denotes transposition,  $i = 1, \dots, 5$  and  $j = 1, \dots, 4$ .

Two modal shapes are considered well correlated if the MAC value is close to one and uncorrelated if it is close to zero.

The matrix of MAC values, reported in Table 5, shows that the first flexural mode is correlated with the first two experimental modes, the natural frequencies of which differ of about 2%. Therefore, both these experimental frequencies can be associated to the first natural mode. Their slight difference denotes an unsymmetrical transversal dynamic behavior, probably because the fibers are not uniformly distributed in all directions along the test object. In the following, the lower of these two frequencies will be assumed as the first experimental one.

Furthermore Table 5 points out that the second flexural mode corresponds to the fourth experimental mode, the third flexural mode to the fifth experimental mode, while the fourth flexural mode is far beyond the investigated frequency range.

Figures 6 and 7 show the first three flexural numerical and experimental modal shapes respectively. It can be noted that they appear almost similar.

The correlation among the first three flexural numerical and experimental modal shapes is highlighted in Figures 8-10, where the circles, which relate to specific degrees of freedom, lie close to a straight line having a slope of about 45°.

The experimental frequencies are reported in Figure 11 against the correlated numerical ones. It

Table 5. Matrix of MAC values.

Numerical mode number	Experimental mode number				
	1	2	3	4	5
1	<b>0.98</b>	<b>0.97</b>	0.08	0.01	0.13
2	0.01	0.01	0.06	<b>0.86</b>	0.03
3	0.10	0.12	0.01	0.00	<b>0.91</b>
4	0.00	0.00	0.01	0.00	0.01

can be noted that the degree of correlation decreases as the order of the frequency increases. In fact, the circles are close to a line with a slope remarkably less than  $45^\circ$ . The correlation between the first numerical and experimental frequencies is very good, the former being only about 5% greater than the latter, but the second and third numerical frequencies are about 19% and 51% greater than the experimental ones. This points out that the finite element model, based on the material properties reported in Section 2, should be updated.

## 5 CONCLUSIONS

In this paper a free vibration testing procedure for a CRP thin-walled cylindrical pole has been presented.

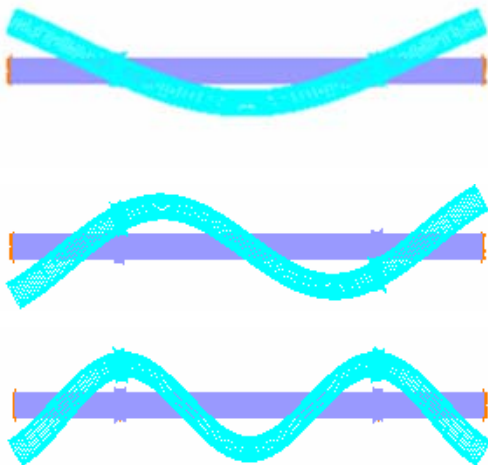


Figure 6. First three flexural numerical modal shapes.

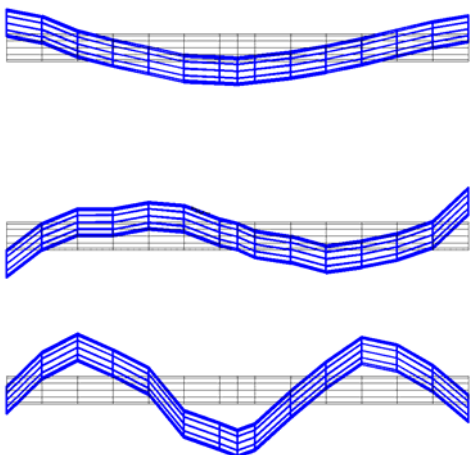


Figure 7. First three flexural experimental modal shapes.

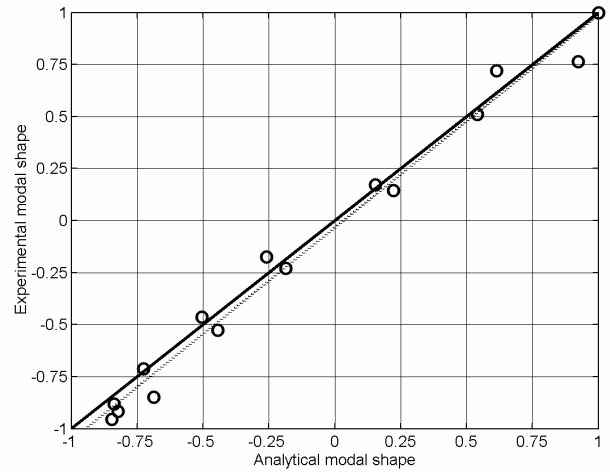


Figure 8. First flexural numerical and experimental modal shapes.

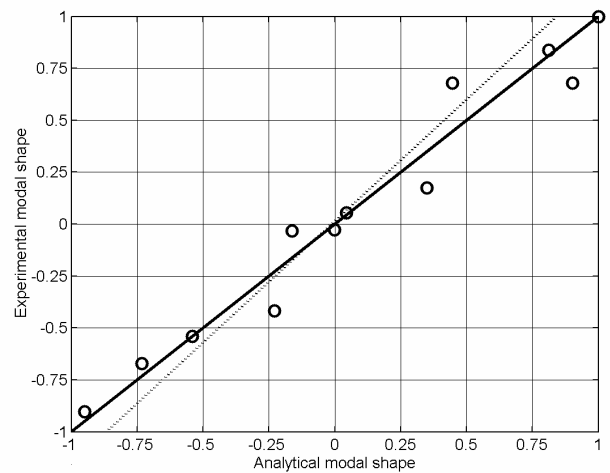


Figure 9. Second flexural numerical and experimental modal shapes.

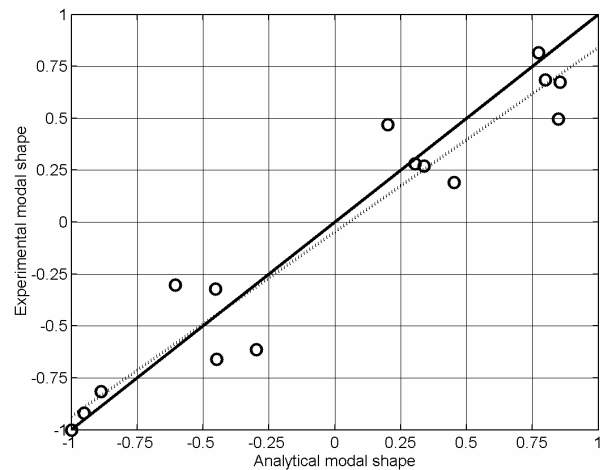


Figure 10. Third flexural numerical and experimental modal shapes.

The dynamic tests have been carried out in “free-free” boundary conditions and the impulse forces have been applied by an impact hammer. The Ibrahim time domain identification technique has been applied in order to estimate the relevant modal properties of the pole. These parameters have been correlated and compared with the corresponding ones ob-



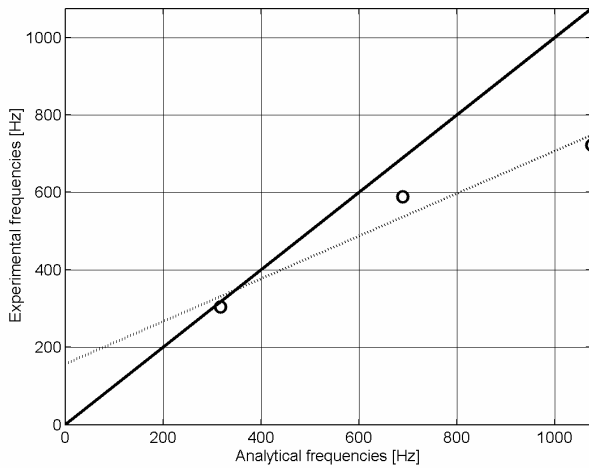


Figure 11. Correlated pairs of natural frequencies.

tained by means of a FE model, formulated on the basis of material properties already evaluated in previous works.

The comparison between the measured parameters and the FE solution shows a good agreement between the fundamental numerical and experimental natural frequencies and between the first three couples of flexural modal shapes. On the contrary the second and the third numerical frequencies are remarkably greater than their experimental counterparts. Therefore further investigations need to be carried out, concerning both numerical modeling and experimental test procedures.

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